Experiment 8

The Pendulum

8.1 Objectives

- Investigate the functional dependence of the period (τ) of a pendulum on its length (L), the mass of its bob (m), and the starting angle (θ_0). The Greek letter tau (τ) is typically used to denote a time period or time interval.
- Use a pendulum to measure g, the acceleration due to gravity.

8.2 Introduction

Everyday we experience things moving in a periodic manner. For example, when you go to a park, you can see children playing on a swingset. As they move back and forth, they are undergoing periodic motion, much like that of a pendulum. Pendula are great tools for measuring time intervals accurately, but they also can be used to measure gravity if you know how. Today, we will investigate how a pendulum works, what affects it period, and try to measure gravity (once again) using what we know in physics.

8.3 Key Concepts

As always, you can find a summary on-line at Hyperphysics[1]. Look for keywords: angular acceleration, pendulum

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

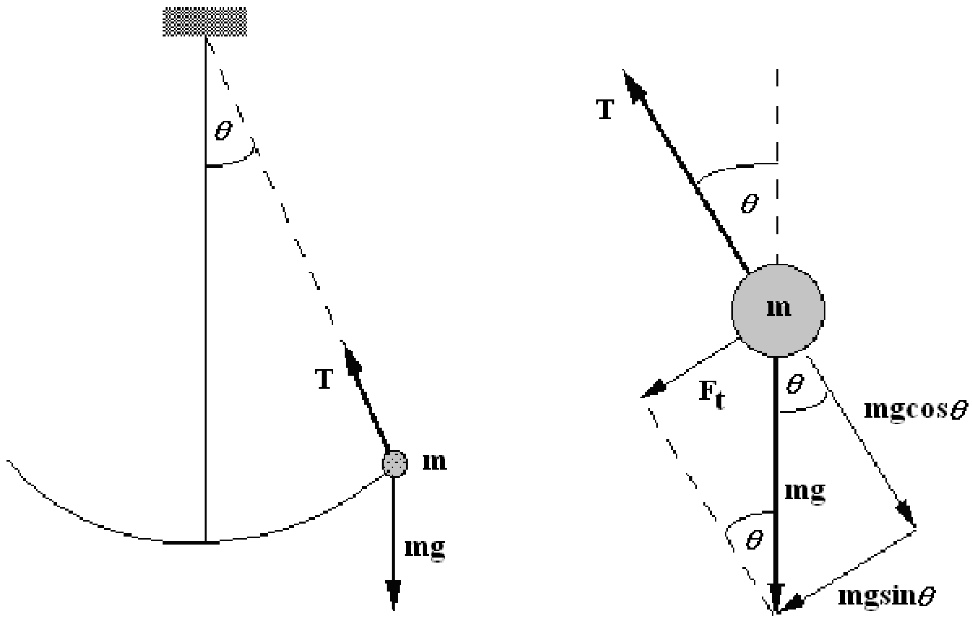

Figure 8.1: Force diagram of a pendulum

8.4 Theory

In the analysis of the motion of a pendulum we should realize that

1. The motion is part of a circle so angular acceleration (α) is a useful variable
2. The angular acceleration will not be a constant throughout the motion

Consider the pendulum shown in Figure 8.1. The weight at the end of the string is called the "bob" of the pendulum. The acceleration, a_t, of the bob **tangent** to the arc "drawn" by the pendulum as it swings is determined by F_t, the force tangent to the arc. Since the tension in the string (T) always acts along the **radius**, it does not contribute to F_t. Decomposing the gravitational force mg into components perpendicular and parallel to the string as shown in Figure 8.1, we find that

$$F_t = mg\sin\theta$$

Therefore the acceleration tangent to the circle is given by:

$$a_t = \frac{F_t}{m} = g \sin\theta$$

The angular acceleration α is then found by the relationship for circular motion

$$\alpha = -\frac{a_t}{r} = -\frac{g}{L} \sin\theta$$

Thus, as we have suggested, *the angular acceleration α is not a constant* but varies as the sine of the displacement angle of the pendulum.

For small angles (about $\theta < 0.5$ radians) angular accelerations can be shown (with a little calculus which we will skip) to lead to an oscillation of the angle θ by

$$\theta = \theta_0 \cos\frac{2\pi t}{\tau}$$

where θ_0 is the angle at time $t = 0$ (when we release the pendulum), and τ is the period of the motion. The period is the time it takes to complete *one full cycle* of the motion.

The period (τ) of a pendulum *depends only on its length (L) and the acceleration due to gravity (g)*. The period (τ) is independent of the mass of the bob (m) and the starting angle (θ_0). The period of a simple pendulum is given by:

$$\tau = 2\pi\sqrt{\frac{L}{g}} \quad \text{or} \quad \tau = \frac{2\pi}{\sqrt{g}}\sqrt{L}$$

This equation has the same form as the equation of a straight line, $y = mx + b$, with an intercept of zero (i.e. $b = 0$). Notice in this equation, the period (τ) corresponds to y and $\sqrt{L}$ corresponds to x.

8.5 In today's lab

Today we will change various parameters of the pendulum and see how they affect it's period. We will change the

1. Mass
2. Starting angle
3. Length

of the pendulum independently of each other and measure the period. We will then plot our results and see if we can accurately measure gravity using a pendulum.

8.6 Equipment

- Masses
- String
- Photogate
- Compass
- Meter Stick

8.7 Procedure

1. Measure the masses of the point masses provided. Use the six heaviest masses in order to get the most accurate data.
2. Measure the length you will use for the trials where you vary the mass m and starting angle θ_0 of the pendulum and record it in the appropriate cells in your excel sheet.
3. Put the photogate on the `PEND` setting.
4. Choose a starting angle such that $\theta_0 < 30°$ from vertical and record it in your data sheet.
5. Place a mass on the end of your pendulum and record it in your data sheet.
6. Move the pendulum to that angle and release it, allowing it to complete one full oscillation. Record the time shown on the photogate in your data sheet. This is your period τ.
7. Repeat steps 5 and 6 five more times using different masses each time.

8. Now choose a mass to use for the starting angle and length trials, hang it on the end of the pendulum, and record the mass in the appropriate cells in your data sheet.

9. Record 6 periods for trials where you vary your starting angle θ_0. Here you can choose any starting angle as long as θ_0 remains less than 30° from the vertical. Make sure that each starting angle is at least 4° different from any other starting angle you used.

10. Keeping the same mass on the end of the string and starting your pendulum at the same starting angle θ_0 you chose in step 4, measure the period for 6 trials where you change the length of the pendulum and record it in your data sheet. Make sure that there is at least a 5 cm difference between the lengths you choose to use.

11. Using the correct formula and the "fill down" method in excel, calculate $\sqrt{L}$ in your data sheet.

12. Create plots for

    - Period (τ) vs. m (for fixed θ_0 and L)
    - Period (τ) vs. θ_0 (for fixed m and L)
    - Period (τ) vs. $\sqrt{L}$ (for fixed m and θ_0)

    in KaleidaGraph. Use the same scale for the axis displaying period τ. You can determine this scale by selecting the smallest and largest values out of **all** of your trials and using those as your minimum and maximum values respectively for your axis limits. Be sure to fit each plot with a best fit line.

8.8 Checklist

1. Excel Sheets

2. Plot for Period (τ) vs. m

3. Plot for Period (τ) vs. θ_0

4. Plot for Period (τ) vs. $\sqrt{L}$

5. Questions

8.9 Questions

1. Comment on how the period depends on each of the three parameters. The period is independent of a parameter if the slope of your best fit line is consistent with zero.

2. Use the slope of the graph of τ vs. $\sqrt{L}$ to calculate g and its uncertainty.

$$\delta g = 2g\frac{\delta(\text{slope})}{\text{slope}}$$

3. Is your value of g consistent with 981 cm/sec^2? If not, suggest a possible source of error.

4. For one of your measurements, calculate the angular acceleration (α) at the starting angle θ_0, at $\theta_0 = 0$, and at the far end of the swing.